Technical Notes

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Heat Transfer to Turbulent Radial Wall Jets

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Introduction

A TURBULENT radial wall jet is being considered as a method of actively cooling a large centrally obscured primary mirror for a high-energy laser beam expander. Toward this end, an experiment was performed that placed a radial wall jet in the center of a flat plate. Data collected from this experiment were used to determine correlations for heat transfer to the jet as a function of radial coordinates. This paper compares the experimental data to an analysis of the radial wall jet convective cooling based on the Reynolds-Colburn analogy. It is this analysis that will lead to a heat transfer relationship suitable for turbulent radial wall jets over a wide range of parametric validity.

Johnson[1] recently undertook heat transfer measurements on a turbulent radial wall jet. The heat flux at the wall surface was maintained constant to simulate laser heating. The resulting data constitute one of a very few heat transfer data bases for radial wall jets in turbulent flow. Because the correlation equations presented by Johnson were intended for a limited range of application, and because the data are deemed important for the heat transfer community, the present work will reanalyze and correlate the data in order to encompass a wider range of parametric validity. Three elements are involved in the approach: a correlation equation between the local surface friction and the local flow coordinates, empirical relationships for the local flow parameters in terms of the flow geometry, and a relation between the friction coefficient and the heat transfer coefficient, i.e., the Reynolds-Colburn analogy.

A general survey of turbulent wall jets was presented by Launder and Rodi.[2] This survey centers on plane wall jets but some work on impinging radial wall jets was also included. In cooling applications, jet impingement is often used to take advantage of the high heat transfer produced at the stagnation point. Heat transfer decreases rapidly as the jet spreads and slows, with the resulting flow configuration similar to a radial wall jet. However, in this radial jet flow region, few heat transfer data have been recorded with sufficient details to permit a nondimensional reconstruction for the purposes of extrapolation.

The work by Donaldson et al.,[13] though complete with heat transfer data, is difficult to generalize to flow configurations where the flow does not start from an impinging jet. In comparison, the present work used the configuration shown in Fig. 1. This experiment consists of a pressurized circular plenum with a circumferential array of holes through which air flows radially outward, generally in choked flow. The characteristic parameters are the starting radius r_o, the aggregate nozzle area A (or the height of an equivalent circumferential slit L), and the initial jet velocity U_o. Data were collected at distances up to 4800 L. A study by Tanaka and Tanaka[4] used a flow geometry similar to the one presented here. The differences were that their nozzle employed a circumferential slit, the maximum nozzle velocity was 80 m/s, and the maximum distance from the nozzle investigated was 100 L. Tanaka and Tanaka's paper contains experimentally derived flow relationships that will be used in the following analysis.

Heat Transfer Analysis

A key relation that connects measured flow with heat transfer is one between the local friction coefficient and its intrinsic coordinates. Two forms are available for use with wall jets. One is the Blausius relation, essentially extracted from pipe flow data, given by Schlichting[5]:

$$C_f = 0.045[U_m\delta/\nu]^{-0.25} \tag{1}$$

where C_f, U_m, δ, and ν are the local coefficient of friction, peak velocity of the wall jet profile, boundary-layer thickness, and gas kinematic viscosity, respectively.

A number of authors have used this expression in their analyses. However, for a radial jet, Poreh et al.[6] have concluded that Eq. (1) does not apply. In a thorough study, Meyers et al.[7] also concluded that Eq. (1) is not appropriate. Another formula similar to Eq. (1) is given by Sigalla,[8] who derived a correlation equation based on his data on plane turbulent wall jets. Sigalla's equation is

$$C_f = 0.0565[U_m\delta/\nu]^{-0.25} \tag{2}$$

It is this equation that will be used to determine heat transfer coefficients using the empirical flow functions, the Reynolds-Colburn analogy, and the assumption of the turbulent Prandtl number = 1. Sigalla's equation is used in this analysis since no corresponding equation of local friction coefficient is available for the turbulent radial wall jet used here. It is significant that Sigalla's equation differs from the Blausius relation only in coefficient. This suggests that the form of Eq. (2) is correct for this analysis, although the coefficient may produce a small error. The assumption of turbulent Prandtl number = 1, which is correct in the section of the jet dominated by the wall, may not hold true in the jet's outer free shear layer. This as-

Received March 1, 1990; revision received July 13, 1990; accepted for publication July 20, 1990.

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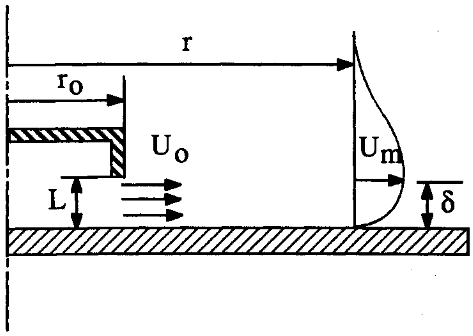

Fig. 1 Wall jet flow geometry and definitions.

sumption may produce an additional source of error in the following analysis.

The flow functions in Eq. (2) come from those obtained by Tanaka and Tanaka,[4] whose relations for U_m and δ are

$$U_m/U_o = 1.52[(r - r_o)/\sqrt{A}]^{-1.09} \tag{3}$$

$$\delta/L = 0.016\ [(r - r_o)/L]^{0.97} \tag{4}$$

where U_o is the initial jet velocity assumed constant over the exit area A, and L is the slit height located at a radial position r_o.

The Reynolds-Colburn analogy can now be formally invoked:

$$St\ Pr^{2/3} = C_f/2 \tag{5}$$

where St is the Stanton number and Pr is the Prandtl number. The Stanton number is defined in terms of local parameters as

$$St = h/\rho C_p U_m \tag{6}$$

where h is the local heat transfer coefficient and C_p is the gas specific heat. Equation (5) implies a global similarity between local flow and heat transfer. The validity of this analogy for a turbulent radial wall jet is best answered by comparing deductions with experimental data. To this end, the Nusselt number and Reynolds number are defined as

$$Nu = hr_o/k \tag{7}$$

$$Re = U_o r_o/\nu \tag{8}$$

where k is the gas thermal conductivity.

Convective heat transfer can now be calculated using Eqs. (2) and (5) and the experimental correlations given by Tanaka and Tanaka[4] [Eqs. (3) and (4)]. This result is

$$Nu/Re^{3/4} = 0.109\ Pr^{1/3}(r_o/L)^{1/4}[(r - r_o)/L]^{-1.06}(L/\sqrt{A})^{-0.818} \tag{9}$$

Results and Discussion

The heat transfer correlation given in Eq. (9) contains some parametric relations that are difficult to verify. For example, the ratio (r_o/L) appears on the right side of Eq. (9) with an exponent of $\frac{1}{4}$ power; this indicates a weak dependence of heat transfer on this variable. Further, the ratio $(L/\sqrt{A})$ should be merged with (r_o/L) since A depends on r_o and L. Equation (9) can now be reduced using the physical parameters of Johnson's experiment (r_o = 7.62 cm, L = nozzle area/nozzle circumference = 0.0103 cm and A = 0.493 cm^2). Substituting these values into Eq. (9) yields (with Pr = 1)

$$Nu/Re^{3/4} = 17.82[(r - r_o)/L]^{-1.06} \tag{10}$$

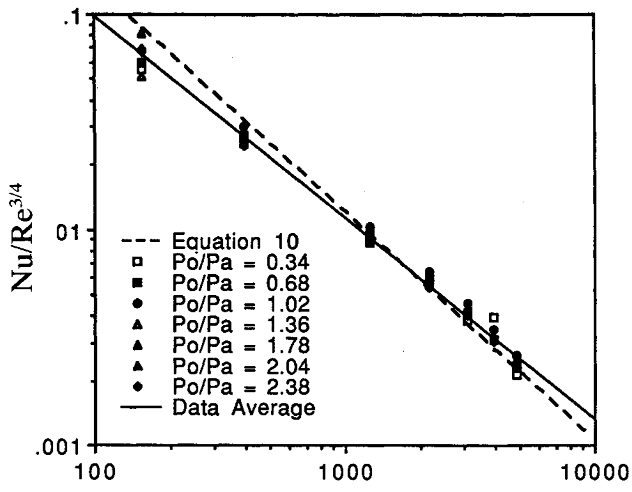

Fig. 2 Comparison of experimental data to Eq. (10).

The data from Johnson's experiment are shown in Fig. 2 together with Eq. (10). The solid line is the least squares curve of best fit through the data points. The experimental variable is the ratio of nozzle gauge pressure to ambient pressure (Po/Pa). It is evident that Eq. (10) agrees well with the experimental data. Note that at low values of r/r_o an appreciable amount of spread occurs in the data, this may be attributable to the developing flow in this region.

Conclusions

The Reynolds-Colburn analogy has been shown to be a valid method of determining heat transfer to a turbulent radial wall jet. Equation (9) was shown to be a good predictor of heat transfer because its derivative expression, Eq. (10), compares with measured data to a confidence level of 30% certainty over the range of r/r_o >1.2 or <9. This error may be attributable to a combination of the use of Sigalla's equation for a plane turbulent wall jet in this radial turbulent wall jet experiment, the assumption of turbulent Prandtl number = 1, or the extrapolation of Tanaka and Tanaka's[4] flow relations beyond their measured data.

Acknowledgments

David Johnson would like to thank Tim Stephens and Mike Lànguirand of the Massachusetts Institute of Technology, Lincoln Laboratory for their assistance. Lit Han would like to thank Denny Aspinwall of Lockheed Missiles and Space Company for bringing about this cooperative effort between Lockheed and Lincoln Laboratory.

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Thermal Charging and Discharging of Sensible and Latent Heat Storage Packed Beds

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Nomenclature

c_p = specific heat at constant pressure, J/kg.K
h_{sf} = specific latent heat of fusion, J/kg
k = thermal conductivity, W/m.K
L = length of the packed bed, m
P = pressure, N/m^2
R = gas constant for Refrigerant-12, J/kg.K
T = temperature, K
t = time, s
μ = absolute viscosity, kg/m.s
ρ = density, kg/m^3

Subscripts
in = inlet
o = initial
s = solid
v = vapor

Introduction

Contemporary applications of packed beds include the use of encapsulated phase change materials as energy storage media for large energy storage densities. The transient response of sensible heat storage packed beds with incompressible transport fluid has been studied in investigations such as those reported by Riaz,[1] Beasley et al.,[2] and Gross et al.[3] Similar studies for packed beds with encapsulated phase change material (PCM) as storage medium have been reported by Pitts and Hong,[4] and Ananthanarayanan et al.[5] A model for transport phenomena in sensible heat storage packed beds with compressible working fluid, modeled as an ideal gas, has been reported by Vafai and Sozen.[6]

In the present work, the thermal energy storage characteristics of a sensible heat storage and a latent heat storage packed bed are investigated. The energy transporting fluid is Refrigerant-12 which is modeled as an ideal gas. Local thermal equilibrium assumption is not used, and the inertia effects are considered in the vapor phase momentum equation.

Received May 29, 1990; revision received September 24, 1990; accepted for publication September 26, 1990.

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Analysis

The physical system considered consists of a horizontal channel filled with randomly packed fixed particles of regularly sized and shaped spheres. Initially the void volume of the packed bed is filled with quiescent working fluid which is at uniform temperature and pressure and in thermal equilibrium with the bed particles. Vapor from a reservoir at a higher temperature and pressure is allowed to flow through the packed bed in the thermal charging mode. In the thermal discharging mode, vapor at a lower temperature is considered to flow through the packed bed.

The governing equations for the present problem were developed by use of the well-known "local volume averaging" technique and they follow from the previous work of Vafai and Sozen.[6] It should be emphasized that in modeling the physical phenomena, the thermophysical properties of the encapsulation material were assumed to be essentially the same as those of the PCM in the case of latent heat storage packed bed. Due to the insulated boundary conditions employed in this investigation, the problem essentially reduces to be one dimensional. The governing equations are given in Ref. 6. The model developed forms a system of five equations in five unknowns.

The initial conditions of the problem were as follows:

$$P_v(x,\, t = 0) = P_o$$
$$T_v(x,\, t = 0) = T_s(x,\, t = 0) = T_o$$
$$u_v(x,\, t = 0) = 0 \tag{1}$$

and the corresponding initial values of ρ_v are calculated from the equation of state.

The boundary conditions of the problem were as follows:

$$P_v(x = 0,\, t) = P_{\text{in}}$$
$$P_v(x = L,\, t) = P_o$$
$$T_v(x = 0,\, t) = T_{v\text{in}} \tag{2}$$

where P_o = 100 kPa, P_{in} = 106.83 kPa, T_0 = 300 K and $T_{v\text{in}}$ = 350 K for thermal charging mode, and $T_{v\text{in}}$ = 300 K for thermal discharging mode. The particle diameter was taken to be 2 mm and the average porosity of the packed bed was taken to be 0.39. The nominal particle Reynolds number for these boundary conditions was 1000.

The modeling of the effective thermal conductivities, the fluid-to-particle heat transfer coefficient, the permeability, and the physical and geometric parameters in the vapor phase momentum equation has been explained in detail in Ref. 6.

The thermophysical properties used in the present investigation were as follows:

Refrigerant-12	*1% Carbon-steel*	*Myristic acid (PCM)*
c_p = 710 J/kg.K	c_p = 473 J/kg.K	c_p = 1590 J/kg.K (solid)
k = 0.0097 W/m.K	k = 43 W/m.K	c_p = 2260 J/kg.K (liquid)
μ = 12.6 × 10^{-6} kg/m.s	ρ = 7800 kg/m^3	k = 0.1 W/m.K
		ρ = 860 kg/m^3
R = 68.7588 J/kg.K		h_{sf} = 200.5 × 10^3 J/kg

The numerical solution scheme for the sensible heat storage packed bed (SHSPB) is detailed in Ref. 6 and is used in the present work too. It should be noted that for the solution of the problem for latent heat storage packed bed (LHSPB), the solution algorithm is the same as for the SHSPB except that during the thermal charging process, once the temperature of the PCM reaches its melting temperature, it remains constant until melting of the PCM is complete. This condition has been incorporated into the numerical algorithm such that at a given point once the melting temperature is reached, the PCM temperature is kept constant while the net rate of heat flowing